In this thesis we study the problem of crack growth on highly heterogeneous periodically distributed composites. The material components are assumed to be linearly elastic, isotropic and homogeneous. Depending on the length scale at which the problem is viewed, several points of view can be adopted. The simplest approach is given by looking at the macroscale. It is assumed that the propagation occurs on a homogeneous material and we are within the scope of linear elastic fracture mechanics. Predicting the behaviour of a growing crack then involves solving the elasticity problems for the cracked structure as the propagation occurs and having a fracture criterion available.

Instead, when the scale of the constituents is considered, the composites are seen to have complex microstructures. This microscopic point of view gives rise to complicated elasticity problems that can be simplified by employing homogenisation. The heterogeneous material is replaced by a homogenised material that has an equivalent behaviour on the macroscopic scale. Mathematically this means that the highly oscillatory coefficients that characterise the behaviour of the heterogeneous material are replaced by constant coefficients.

Homogenisation allows us to look at a heterogeneous composite at the macroscale as if it were homogeneous. This means that we have less computational complexity but also less accuracy. As the purpose of this thesis is to look at the problem of fracture from the scale of the constituents, homogenisation per se is not an option. However, the prohibitive complexity of elasticity problems for composites has to be addressed somehow, and so we look for other techniques. Domain decomposition methods can be considered as a valid alternative. They solve the original problem by dividing the computational domain in smaller subdomains. This divide and conquer logic certainly looks promising, even allowing parallel implementations, but it requires the microscale to be resolved throughout which seems unnecessary. Indeed, when a crack propagates, we focus on the crack tip and its vicinity. Elsewhere the heterogeneous nature of the material does matter, but it is not as relevant.

This leads us to consider a hybrid approach combining homogenisation and domain decomposition techniques. The idea is to look at the problem of crack propagation at a microscopic length scale only in the vicinity of the crack tip and at the macroscale everywhere else. This approach makes the problems of elasticity of periodically distributed composites feasible and we show that it provides the desired microscopic accuracy. In particular, with this technique we are able to set up a procedure to predict the path that a crack propagating on a layered composite will follow. Naturally, the interaction of the crack with the material interfaces also has to be addressed.